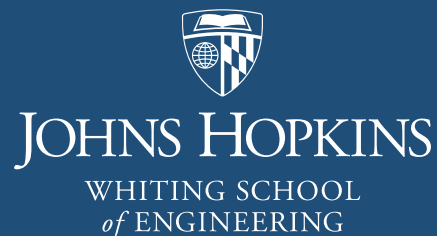# Large-eddy Simulation of Separated Flows Using a New Integral Wall Model

Francois Cadieux, Xiang I.A. Yang, Jasim Sadique,

Rajat Mittal, Charles Meneveau

AMS Seminar Series

NASA Ames Research Center, June 30, 2016

JOHNS HOPKINS

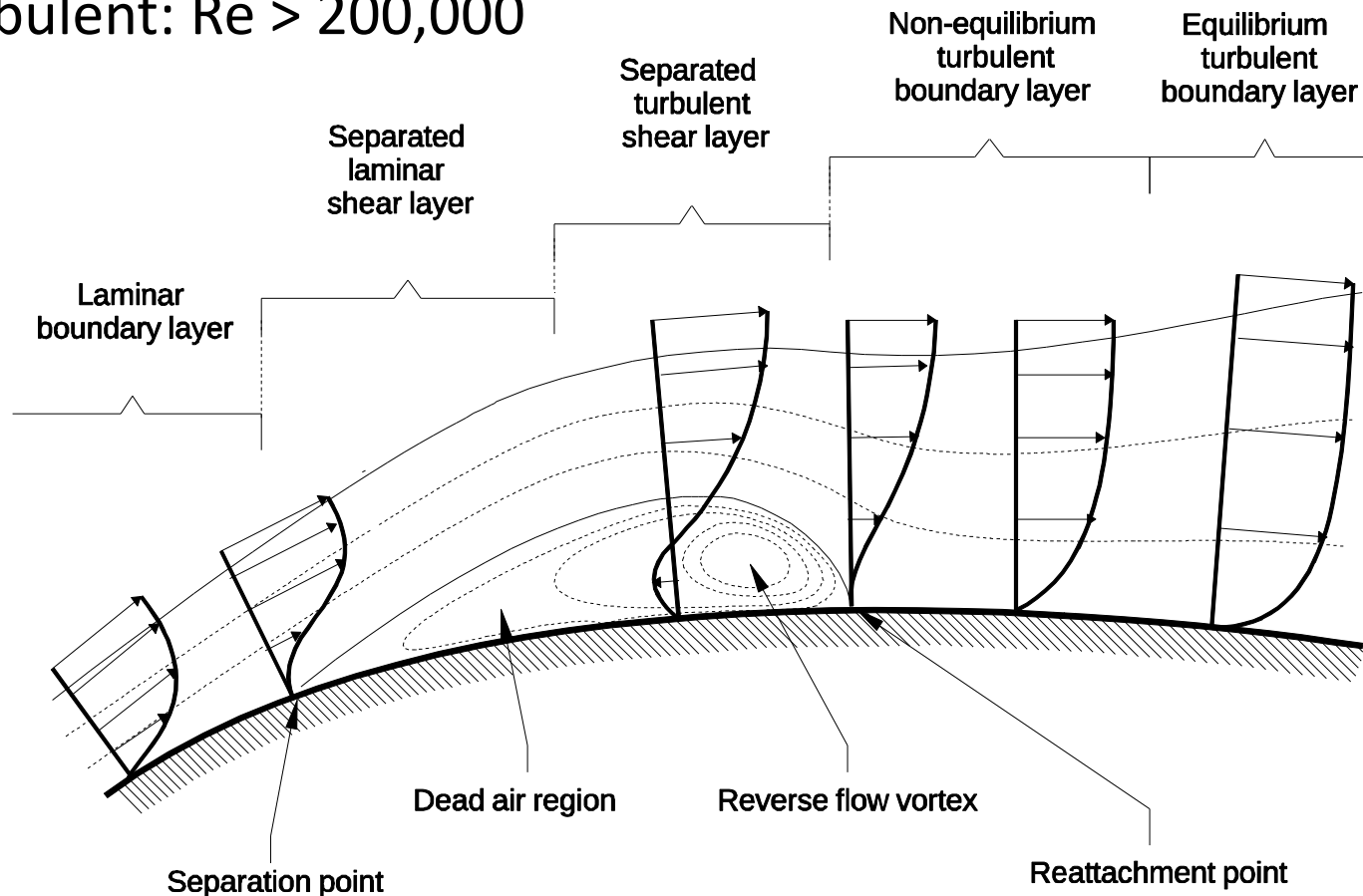WHITING SCHOOL
of ENGINEERING

Motivation

- Separated flows and recirculation regions occur on airfoils and blades for a wide range of Reynolds numbers from $O(10^4)$ to $O(10^6)$

Separation Bubbles

Laminar: 10,000 < Re < 200,000
Turbulent: Re > 200,000

Non-equilibrium
turbulent
boundary layer

Equilibrium
turbulent
boundary layer

Separated
turbulent
shear layer

Separated
laminar
shear layer

Laminar
boundary layer

Dead air region   Reverse flow vortex

Separation point   Reattachment point

Sketch based off Horton H (1968) "Laminar separation bubbles in two and three dimensional incompressible flow", Ph.D. diss., University of London.

Research Goals

- Create predictive simulation tool for separated flows that is:
  - High-fidelity
  - Tractable for high Reynolds number flows

- To enable:
  - Optimization of wing, blade, flap design
  - Rapid testing of active flow control strategies

Why not RANS?

RANS:

- length of recirculation strongly depends on turbulence model

- transition to turbulence is difficult to predict

Spalart, P. and Strelets, M. (2000), "Mechanisms of transition and heat transfer in a separation bubble", *J. Fluid Mech.* **403**, 329.
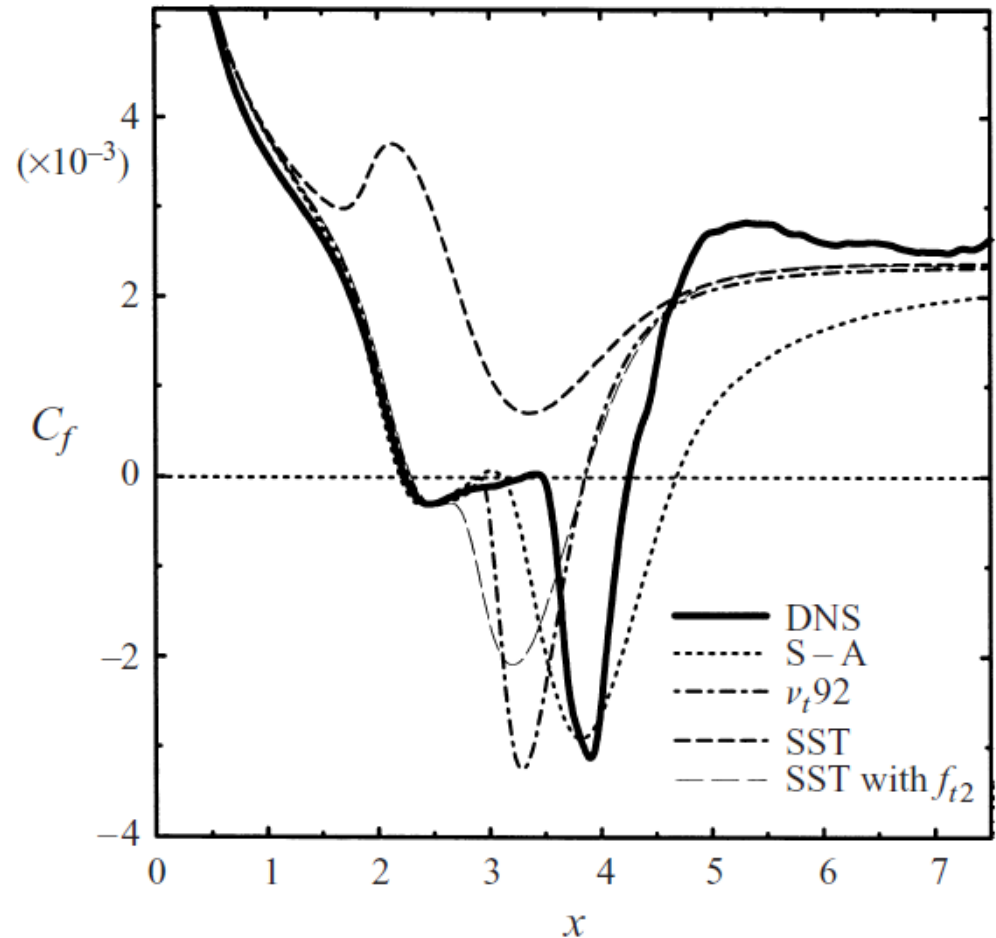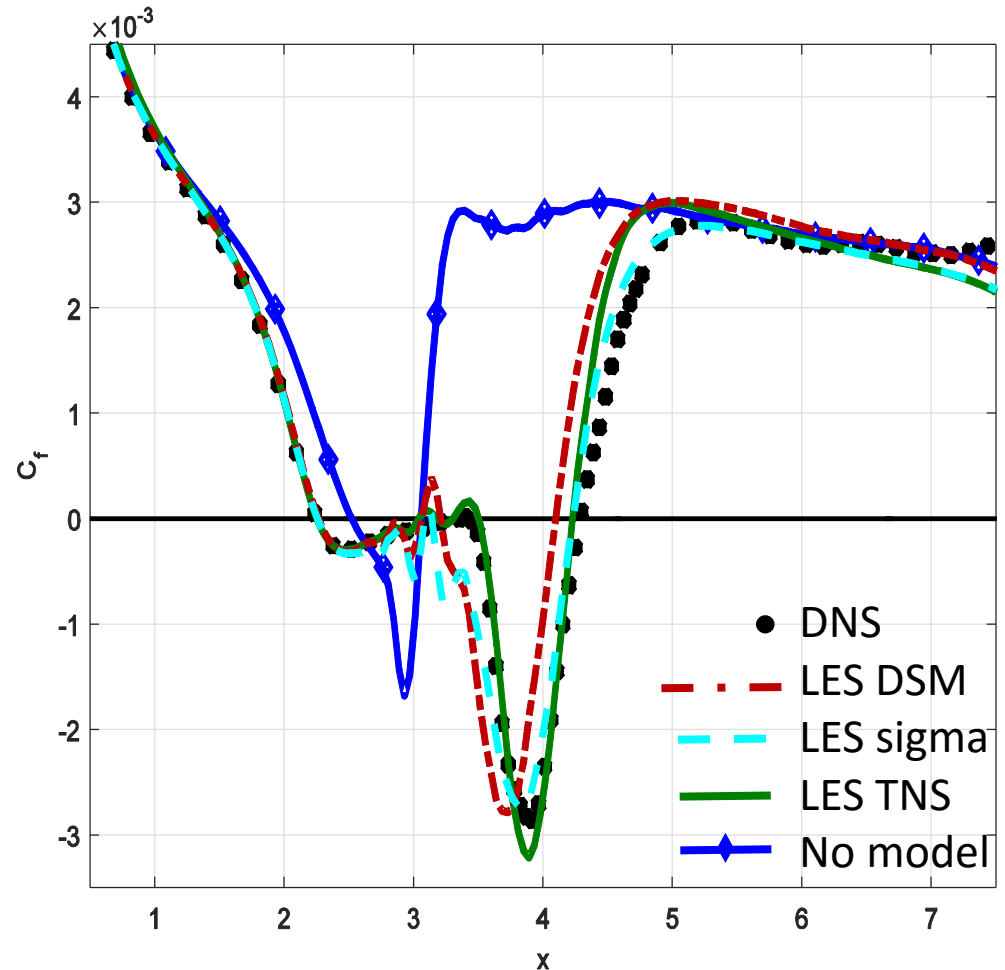
FIGURE 5. Skin-friction coefficients.

5

Why LES?

LES:

- Can capture mean flow, Cp, Cf, and Reynolds stress accurately at resolutions on the order of 1% of DNS

- Largely insensitive to choice of subgrid-scale model

Cadieux, F. and Domaradzki, J. (2015) "Performance of subgrid-scale models in coarse large eddy simulations of a laminar separation bubble", *Phys Fluids,* **27**, 045112

Skin-friction coefficients

Why Wall-Modeled LES?
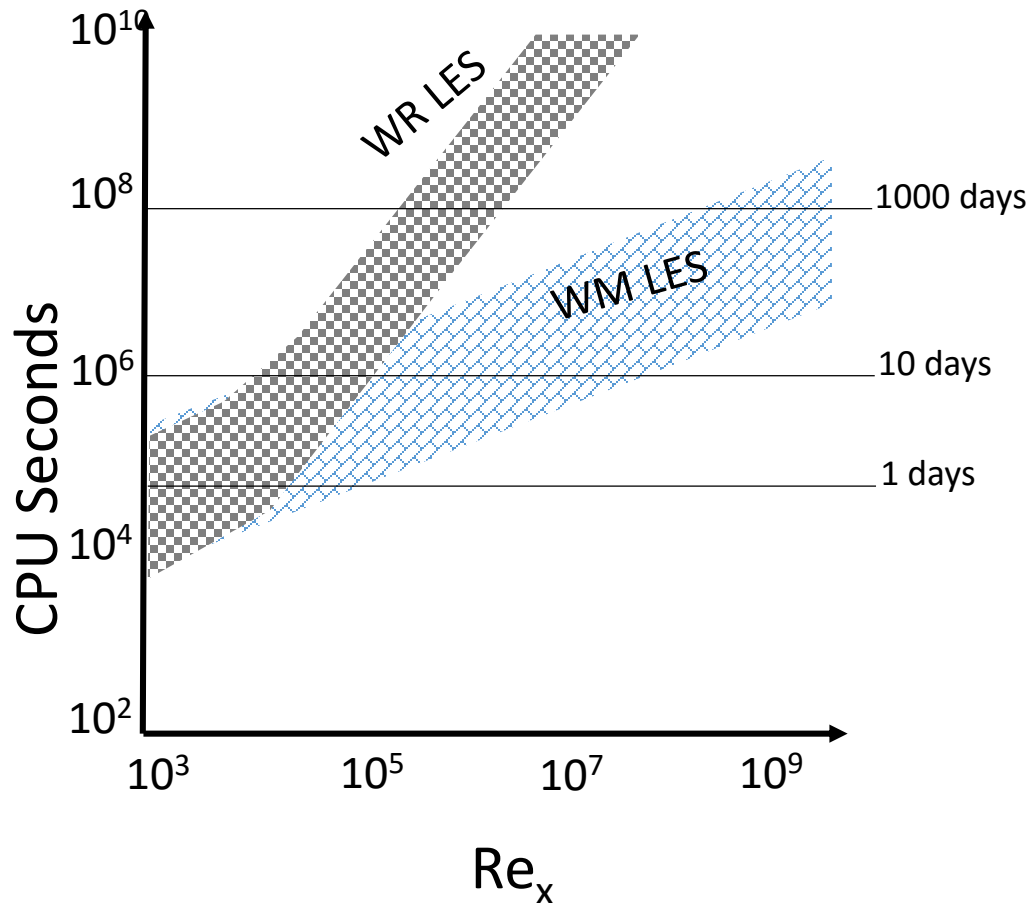
Wall-resolved LES:

- \# of points resolve viscous sublayer:
$$(N_x N_y N_z) \propto Re^{2-\epsilon}$$
$$\epsilon < 0.2$$

- For $Re>10^5$, >90% of grid points are used in <10% of the simulation domain (near boundaries)

Piomelli, U. (2008), "Wall-layer models for large-eddy simulations", *Progress in Aerospace Sciences* **44**, 437.
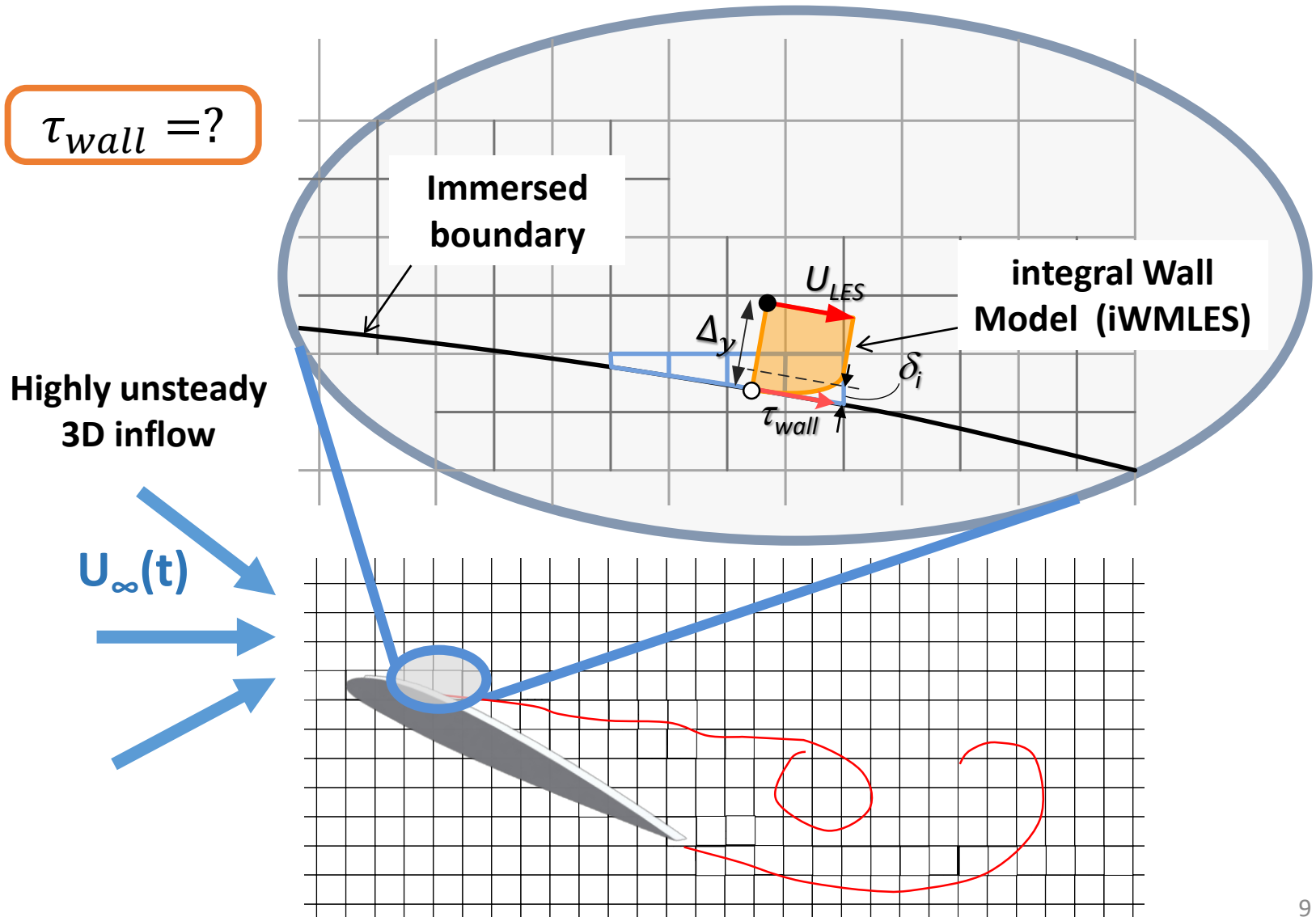
Why Wall-Modeled LES?

Estimates for Canonical Turbulent Boundary Layer

	$Re_x=10^6$	$Re_x=10^7$
Wall Resolved LES	8.7×10^7	1.4×10^{10}
Hybrid RANS-LES	1.4×10^7	2.0×10^7
Integral Wall Model LES*	3.0×10^6	3.0×10^6

Estimated # of grid points in the boundary layer region for different methods and Reynolds numbers.

*Yang, X.I.A., Sadique, J., Mittal, R. & Meneveau, C. (2015), "Integral Wall Model for Large Eddy Simulations of wall-bounded turbulent flows". *Phys. Fluids* **27**, 025112.

What is wall-modeled LES?

$$\tau_{wall} = ?$$

Immersed boundary

U_{LES}

integral Wall Model (iWMLES)

Δ_y

δ_i

Highly unsteady 3D inflow

τ_{wall}

$U_\infty(t)$

LES Wall-modeling approaches

	Equilibrium	Zonal/Hybrid	Dynamic Slip	Integral WM
Solves	Equilibrium TBL (log law)	Full RANS	ODE for slip velocity	Vertically Integrated Momentum
Strength	Simple	Wealth of experience	Simple	Versatile
Weaknesses	Needs correction for laminar/transitional flow	Requires embedded grid and RANS solver	Grid dependence, slip is not physical	Assumed profile may not be valid for all flows
CPU Cost	Negligible	High	Low	Very Low

Integral Wall Model (iWMLES)

Filter velocities in time to match near wall time scale
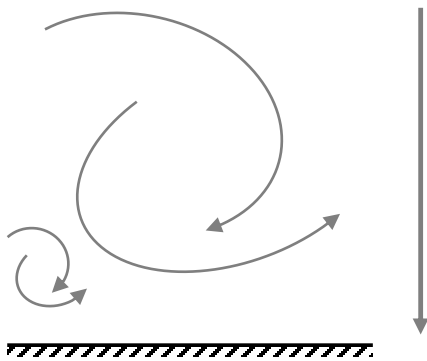
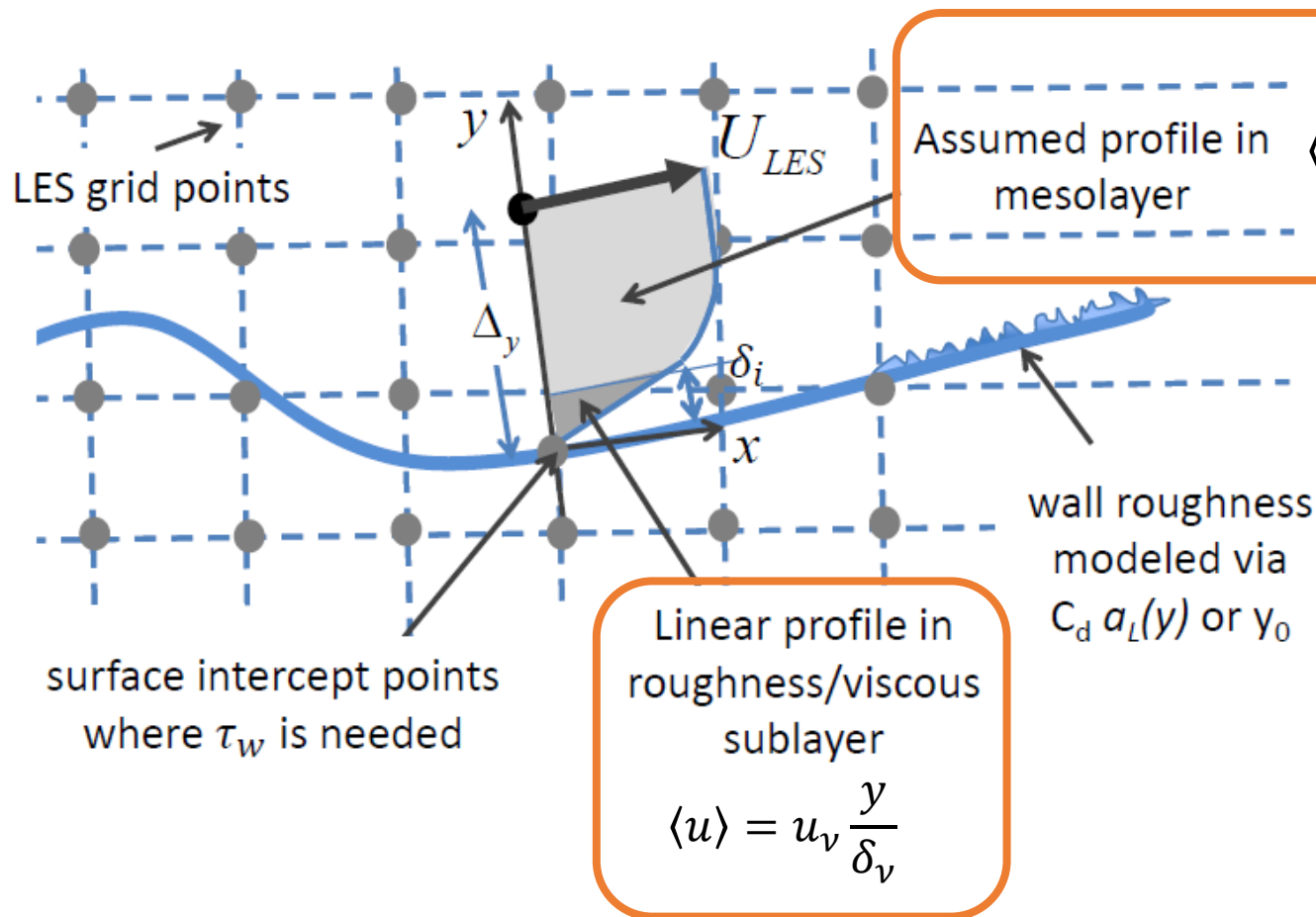$$\langle u_i \rangle = \int_{-\infty}^{t} u_i(x,y,z,t')\frac{1}{T_{wall}}e^{-\frac{t-t'}{T_{wall}}}dt'$$

$$U_{LES} = \int_{-\infty}^{t} u(x,y=\Delta_y,z,t')\frac{1}{T_{wall}}e^{-\frac{t-t'}{T_{wall}}}dt'$$

where $T_{wall} = \frac{\Delta_y}{\kappa u_\tau}$

→ Obtain RANS like equations for $\langle u_i \rangle$ with $\nu_\tau = l_m \left| \frac{\partial \langle U \rangle}{\partial y} \right|$

→ Vertically integrate equations from 0 to Δ_y

→ Solve for τ_w using a parametric velocity profile

Integral Wall Model (iWMLES)

Use von-Karman-Paulhausen's integral method:
Assume velocity profile & integrate BL eqn analytically

LES grid points

U_{LES}

y

Δ_y

δ_i

x

Assumed profile in mesolayer

$$\langle u \rangle = u_\tau \left[C + \frac{1}{\kappa} \log\left(\frac{y}{\Delta_y}\right) + A \frac{y}{\Delta_y} \right]$$

wall roughness modeled via $C_d\ a_L(y)$ or y_0

surface intercept points where τ_w is needed

Linear profile in roughness/viscous sublayer

$$\langle u \rangle = u_\nu \frac{y}{\delta_\nu}$$

12

Integral Wall Model (iWMLES)

Solve for 6 parameters to satisfy 6 constraints (for x):

1, 2) Velocity Continuity: $\langle u \rangle \left(y = \Delta_y \right) = U_{LES} \rightarrow u_\tau (C + A) = U_{LES}$

$$\langle u \rangle (y = \delta_i^+) = \langle u \rangle (y = \delta_i^-) \rightarrow u_\nu \frac{\delta_i}{\delta_\nu} = u_\tau \left[C + \frac{1}{\kappa} \log \frac{\delta_i}{\Delta_y} + A \frac{\delta_i}{\Delta_y} \right]$$

3) Inner Layer Height: $\delta_i = \min \left[\max \left(k, 11 \frac{\nu}{u_\tau} \right), \Delta_y \right]$

4) Inner Length Scale: $\delta_\nu = \frac{1}{u_\nu} \left(\nu + \nu_{\tau, y=0} \right)$

5) Wall shear stress: $\tau_w = u_\tau^2 = u_\nu^2 + \int_0^k C_d a_L \langle u \rangle^2 dy$

6) **Vertically Integrated Momentum Equation:**

$$\frac{\partial}{\partial t} \int_0^{\Delta_y} \langle u \rangle \mathrm{dy} + \frac{\partial}{\partial x} \int_0^{\Delta_y} \langle u \rangle^2 \mathrm{dy} - U_{LES} \frac{\partial}{\partial x} \int_0^{\Delta_y} \langle u \rangle \mathrm{dy} + \frac{1}{\rho} \frac{\partial p}{\partial x} \Delta_y = (\nu + \nu_\tau) \frac{\partial \langle u \rangle}{\partial y} \Big|_{y=\Delta_y} - \tau_w$$

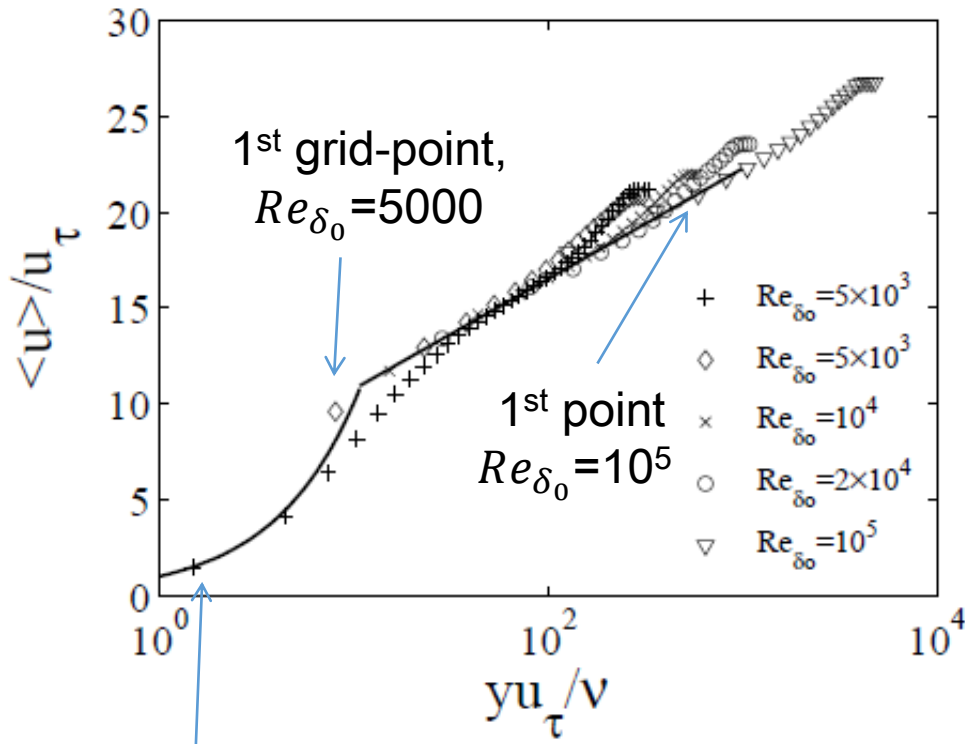Evaluated Analytically

13

Numerical Methods

ViCar3D

- Cartesian finite difference: 2nd order in space and time

- σ-model for subgrid-scale stress term in LES equations

- Recycle-rescale method of *Lund et al.* for developing turbulent boundary layer

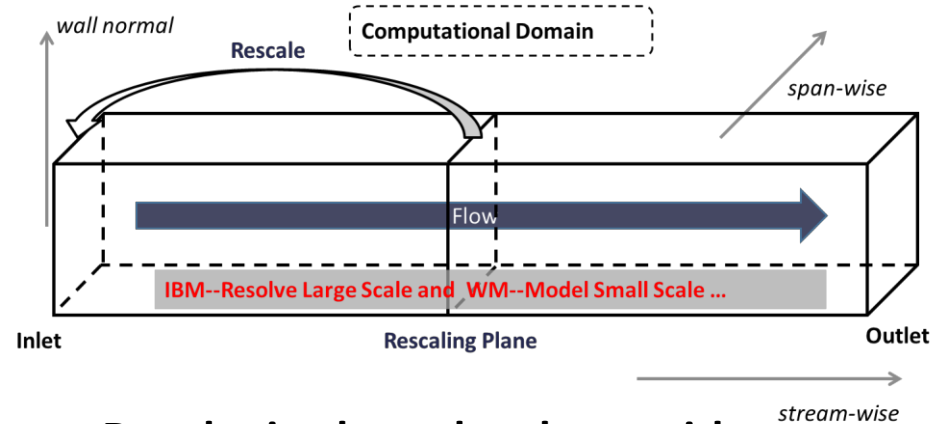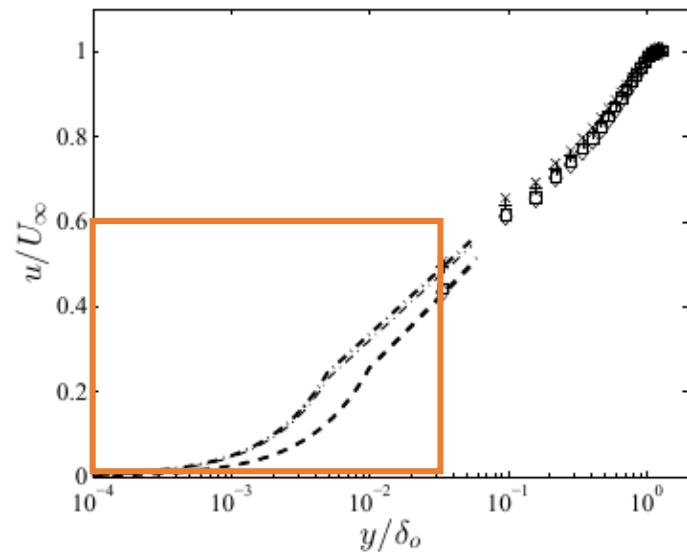- Sharp immersed boundary method

● Fluid-Cell   ◉ Ghost-Cell   ▣ Solid-Cell
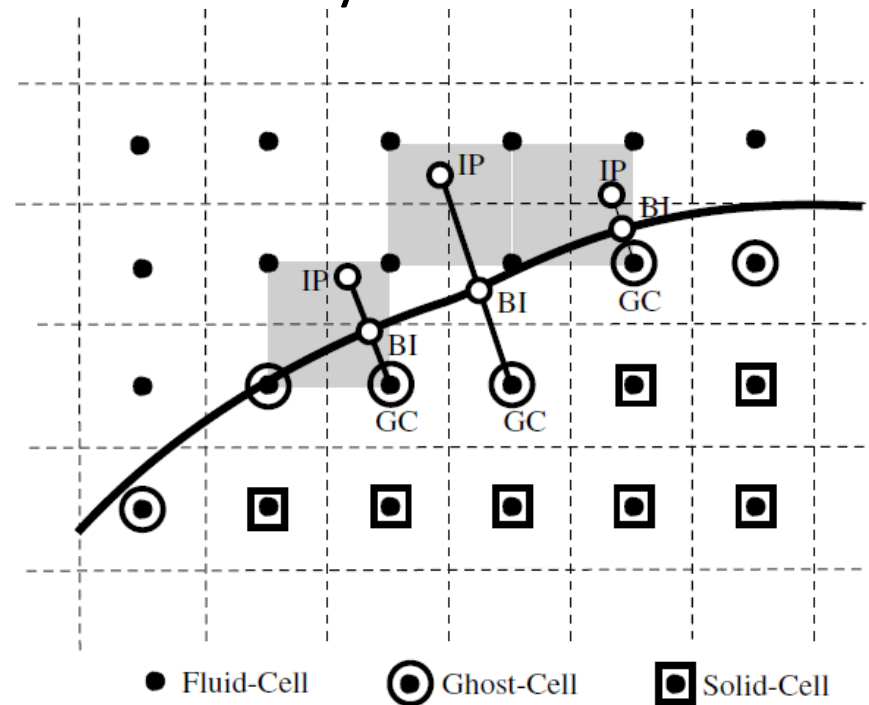
iWMLES Validation I

Flat plate developing boundary layer

1st grid-point, Re_{δ_0}=5000

1st point Re_{δ_0}=10^5

Legend:
- $+$ Re_{δ_o}=5×10^3
- $\diamond$ Re_{δ_o}=5×10^3
- $\times$ Re_{δ_o}=10^4
- $\circ$ Re_{δ_o}=2×10^4
- $\triangledown$ Re_{δ_o}=10^5

1st grid-point, $Re_{\delta_0} = 5000$ ("wall-resolving")

wall normal   Rescale   Computational Domain   span-wise

Flow

IBM--Resolve Large Scale and WM--Model Small Scale …

Inlet   Rescaling Plane   Outlet

stream-wise

Developing boundary layer with unresolved surface roughness

-   k=0.01, 0.005 for Re=2×10^5, 10^6, y_0 = 0.0016, 0.00075;
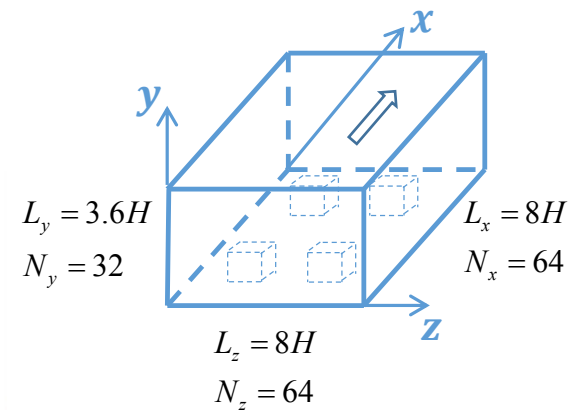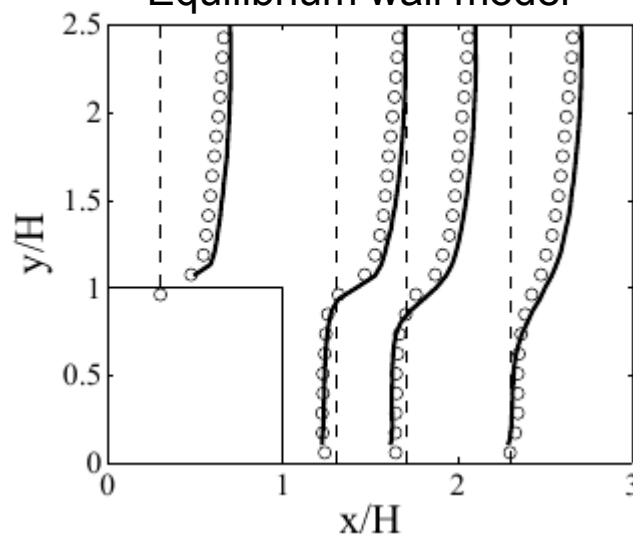
iWMLES Validation II

- i-WMLES

$$U \qquad u_\tau$$

Re$_H$ = 3,800

- Equilibrium wall model

$$U \qquad u_\tau$$
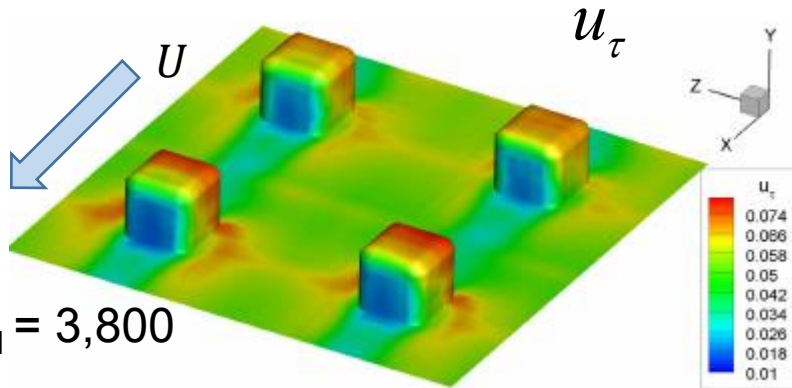
- i-WMLES

- Equilibrium wall model

$L_y = 3.6H$
$N_y = 32$

$L_x = 8H$
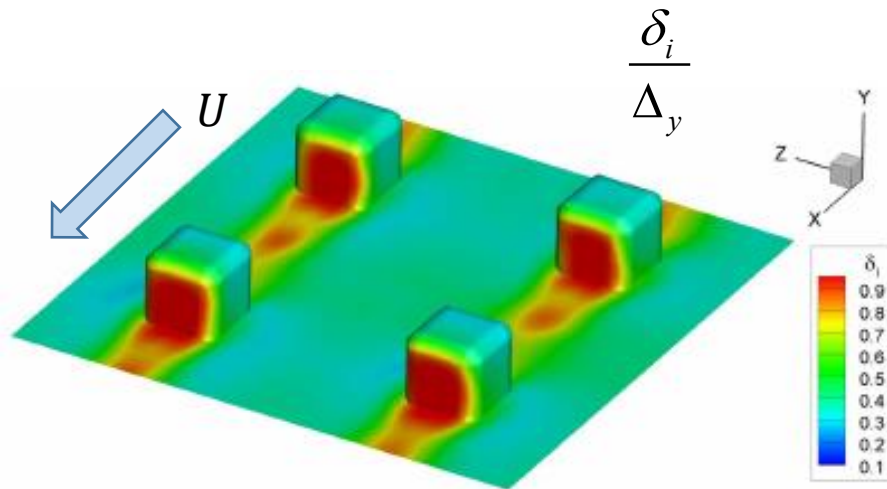$N_x = 64$

$L_z = 8H$
$N_z = 64$
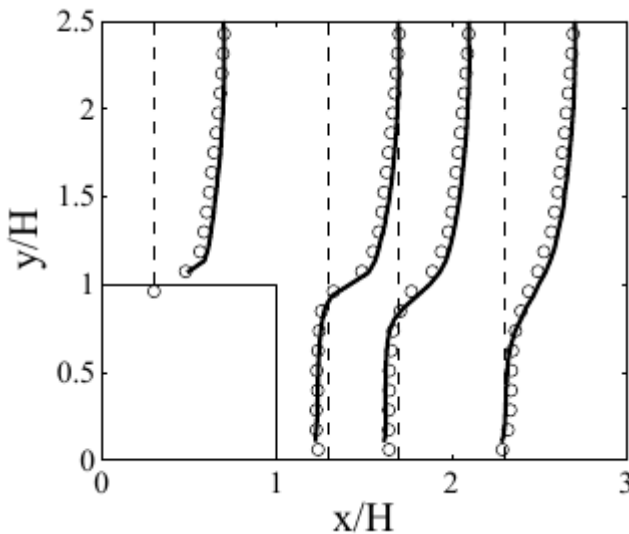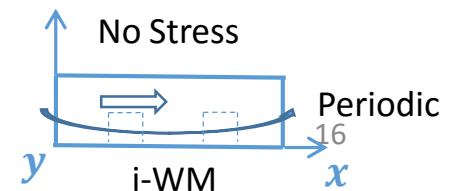
— E. Meinders and K. Hanjalic, "Vortex structure and heat transfer in turbulent flow over a wall-mounted matrix of cubes," International Journal of Heat and Fluid Flow 20, 255 (1999).
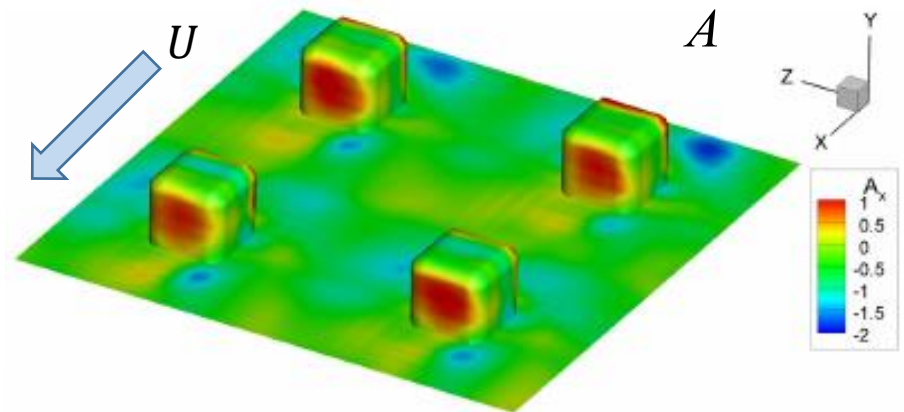
No Stress

Periodic

16

i-WM

iWMLES: Influence of parameters

- Effect of height of linear layer δ_i

$Re_H = 3{,}800$

- Effect of non-equilibrium terms

$$\langle u \rangle = u_\tau \left[C + \frac{1}{\kappa} \log\left(\frac{y}{\Delta_y}\right) + \boxed{A} \frac{y}{\Delta_y} \right]$$
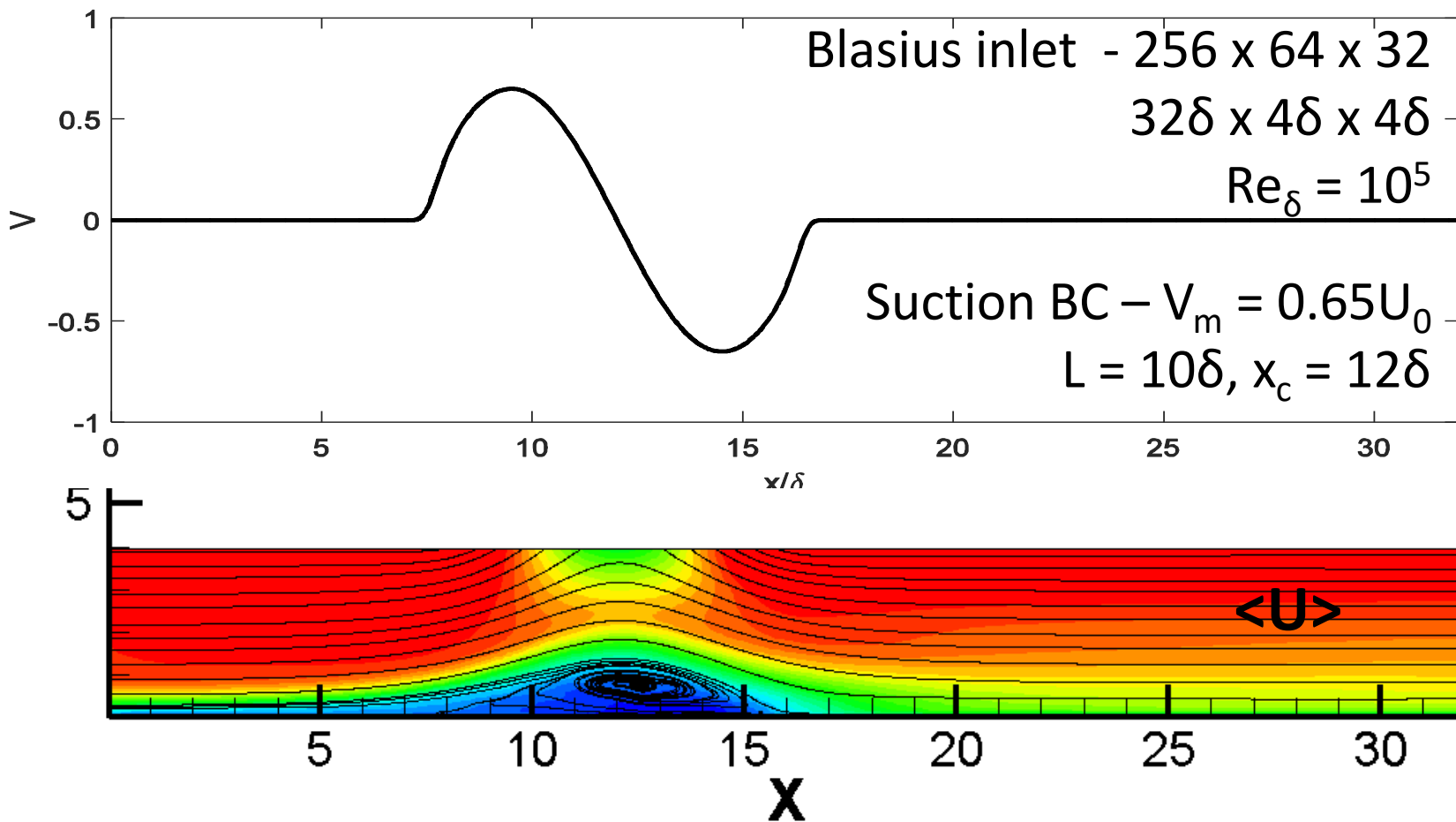

$\dfrac{\delta_i}{\Delta_y}$

U

A

U

Yang, X.I.A., Sadique, J., Mittal, R. & Meneveau, C. (2015), "Integral Wall Model for Large Eddy Simulations of wall-bounded turbulent flows". *Phys. Fluids* **27**, 025112.

Specific Objectives

- Demonstrate that iWMLES can predict transition to turbulence and separation
  - Laminar separation bubble application

- Validate integral Wall Model (iWMLES) for separated flows at high Re against wall-resolved LES
  - Create benchmark wall-resolved LES
  - For the same grid except near wall, compare Cf, Cp
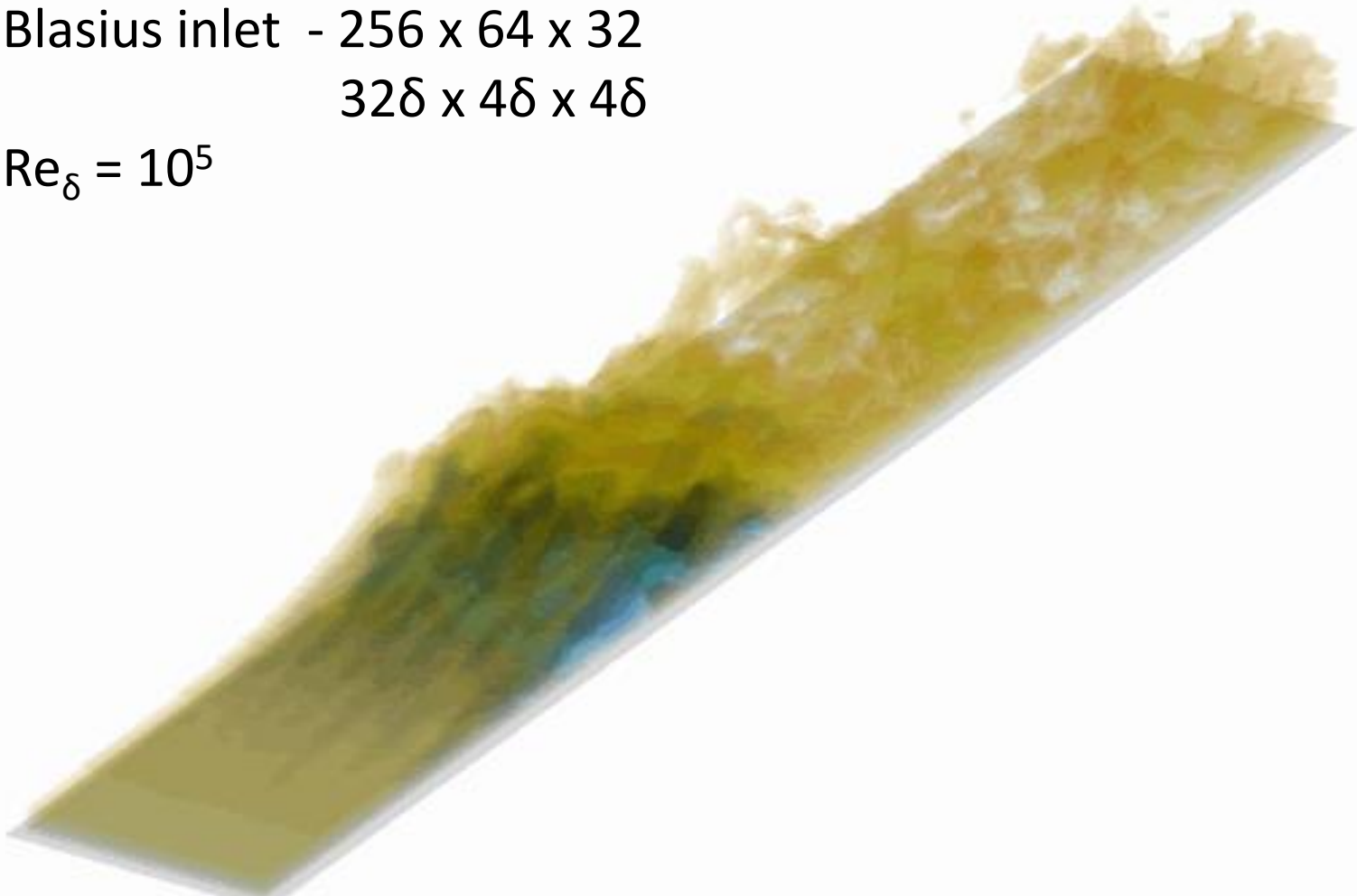
Flow over flat plate with suction boundary condition

Blasius inlet - 256 x 64 x 32

$32\delta \times 4\delta \times 4\delta$

$Re_\delta = 10^5$

Suction BC $- V_m = 0.65U_0$

$L = 10\delta$, $x_c = 12\delta$

Results: Laminar Separation Bubble

Instantaneous streamwise velocity

Blasius inlet - 256 x 64 x 32
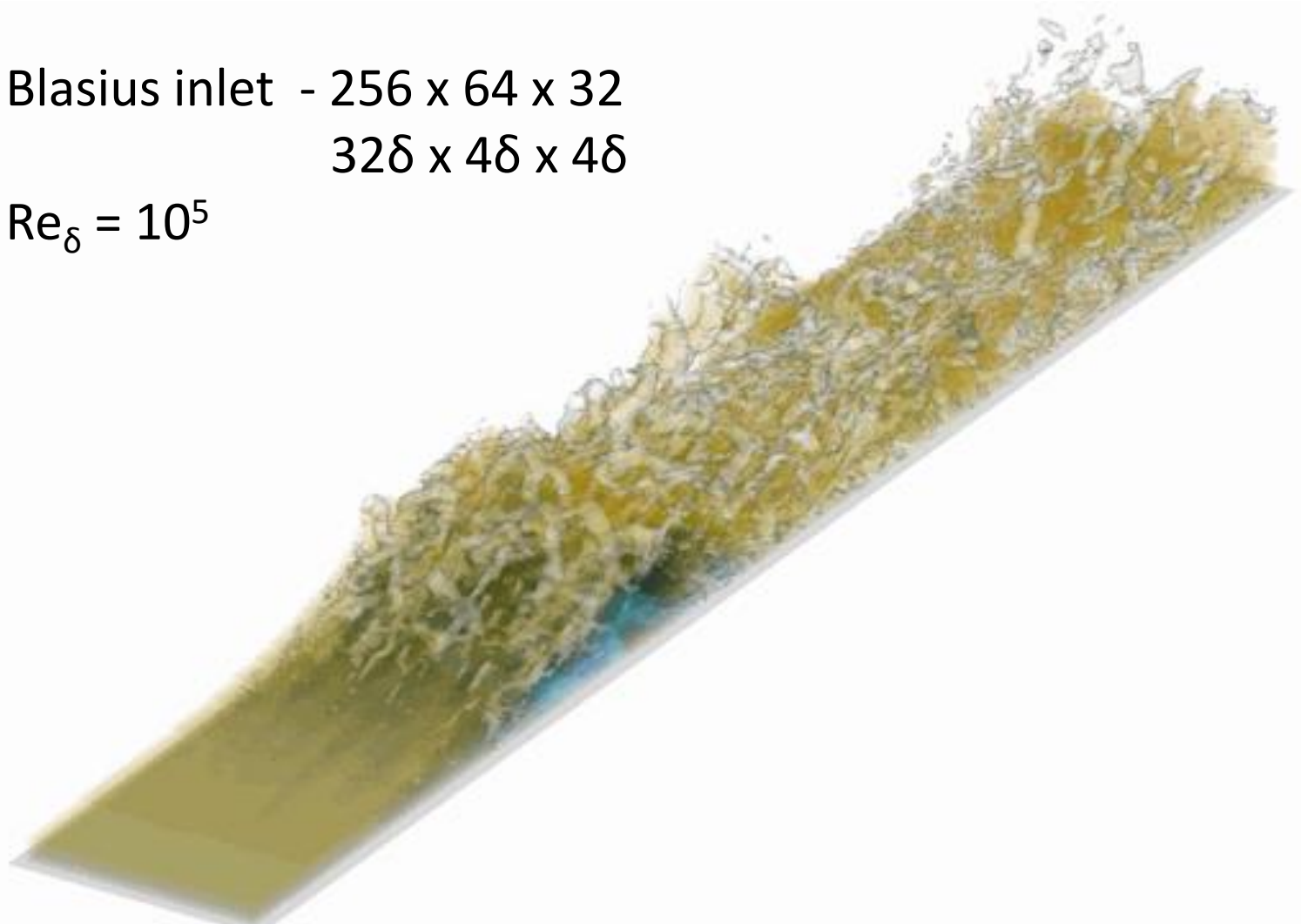
32δ x 4δ x 4δ

$Re_\delta = 10^5$

Results: Laminar Separation Bubble

Instantaneous U with iso-surfaces of Q-criterion

Blasius inlet - 256 x 64 x 32
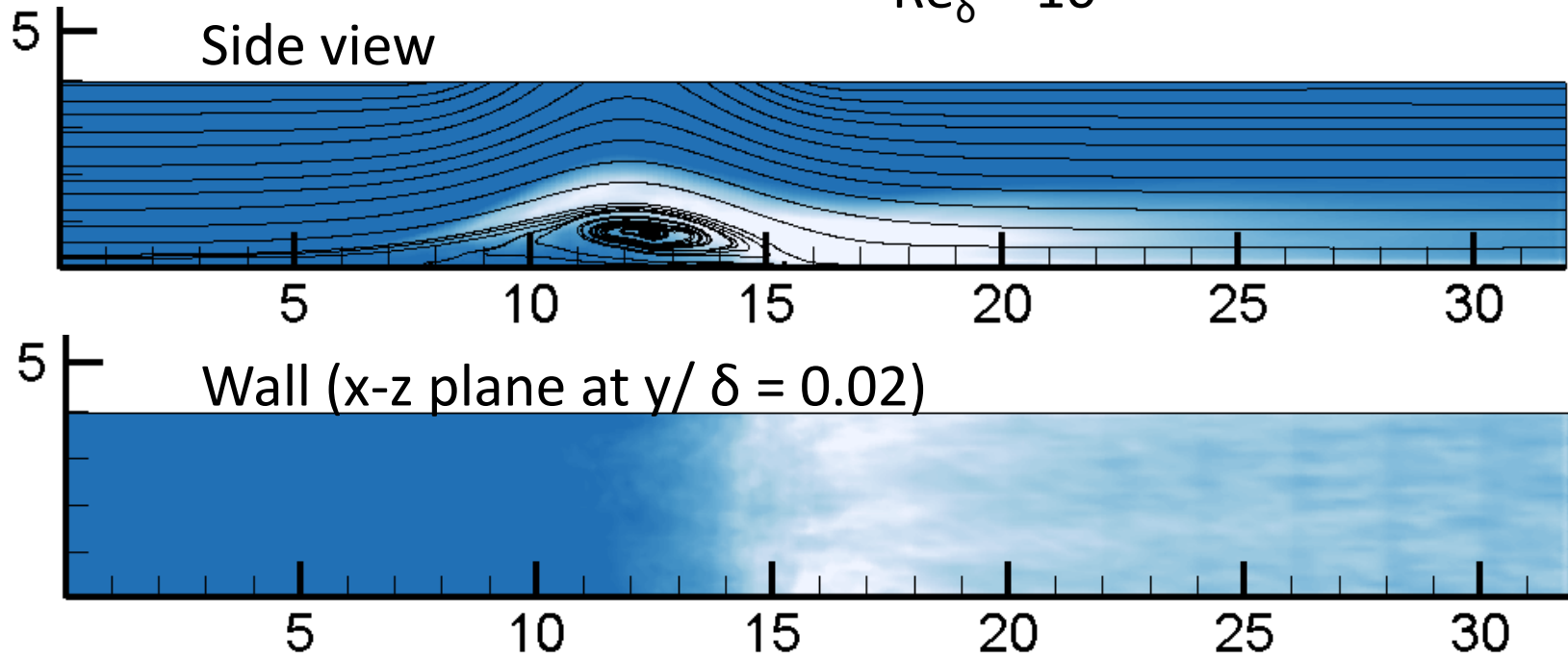
32δ x 4δ x 4δ

$Re_{\delta} = 10^5$

Turbulent Kinetic Energy

Blasius inlet - 256 x 64 x 32

32δ x 4δ x 4δ

$Re_\delta = 10^5$

Side view

Wall (x-z plane at $y/\delta = 0.02$)

Turbulent flow over flat plate with suction BC

$Re_\delta = 16{,}000$

$$v(x, 6\delta) = 0.6 \exp\left(-6\left(\frac{2(\mathrm{x}-27)}{26}\right)^8\right.$$

Recycle-rescale plane

6δ

4δ

5.25δ

26δ

42δ

23
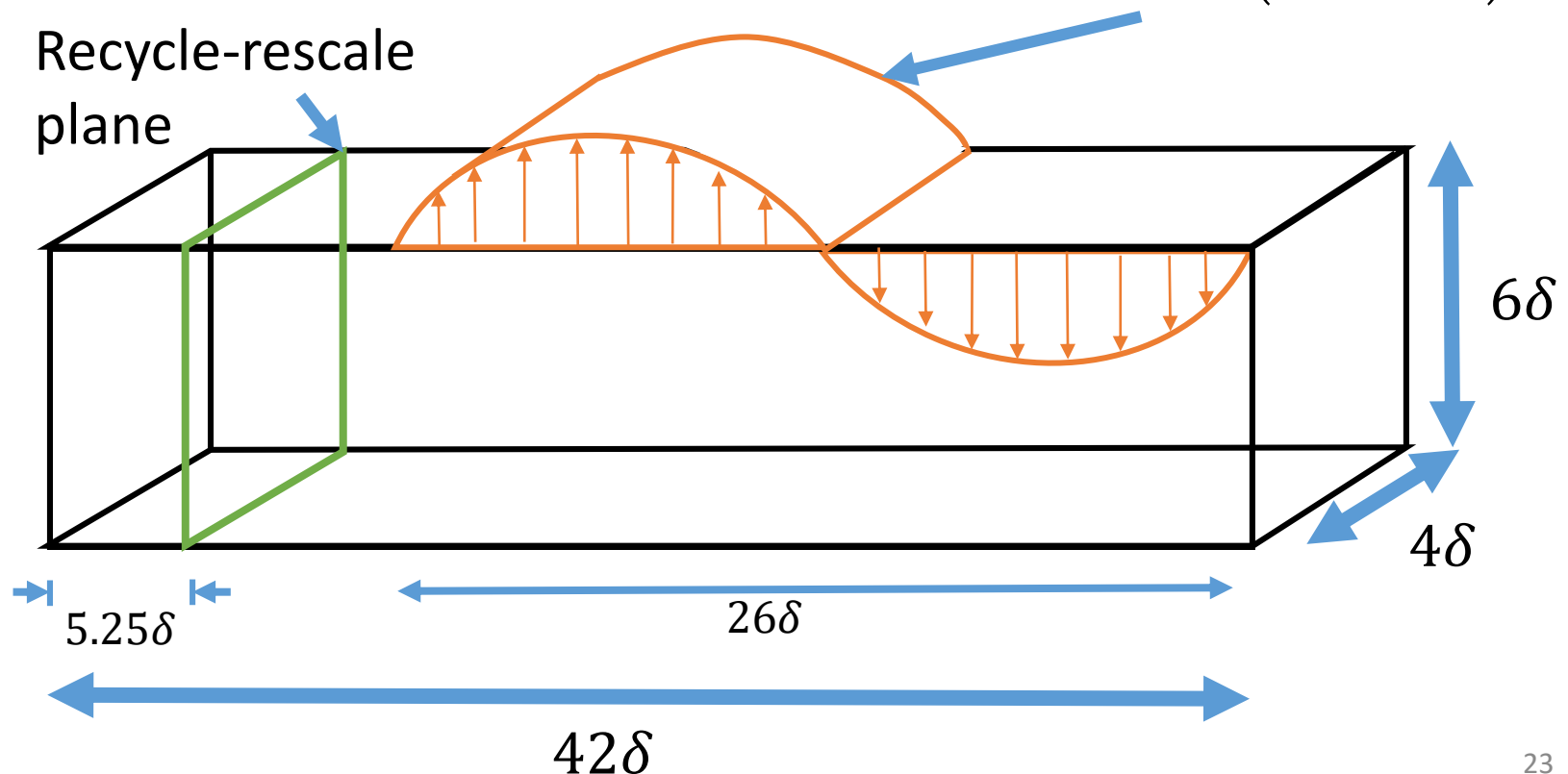
Setup: Turbulent Recirculation Zone

Wall-resolved LES vs iWMLES Resolution

	LES	iWMLES
$N_x \times N_y \times N_z$	$256 \times 128 \times 33$	$256 \times 96 \times 33$
$\Delta x/\delta, \quad \Delta x^+$	0.164, 100	0.164, 100
$\Delta z/\delta, \quad \Delta z^+$	0.125, 75	0.125, 75
$\Delta y/\delta, \quad \Delta y^+$	0.00125, <1	0.05, 16
$\Delta_y, \quad \Delta_y^+$	--	0.175, ~100

$\Delta_y \sim 3\,\Delta y(y=0)$ to avoid feeding the WM the LES under-resolution error in near-wall and to eliminate log-layer mismatch*

*Larsson, J. et al (2016). "Large eddy simulation with modeled wall-stress: recent progress and future directions", *Mechanical Engineering Reviews,* **3**:1.
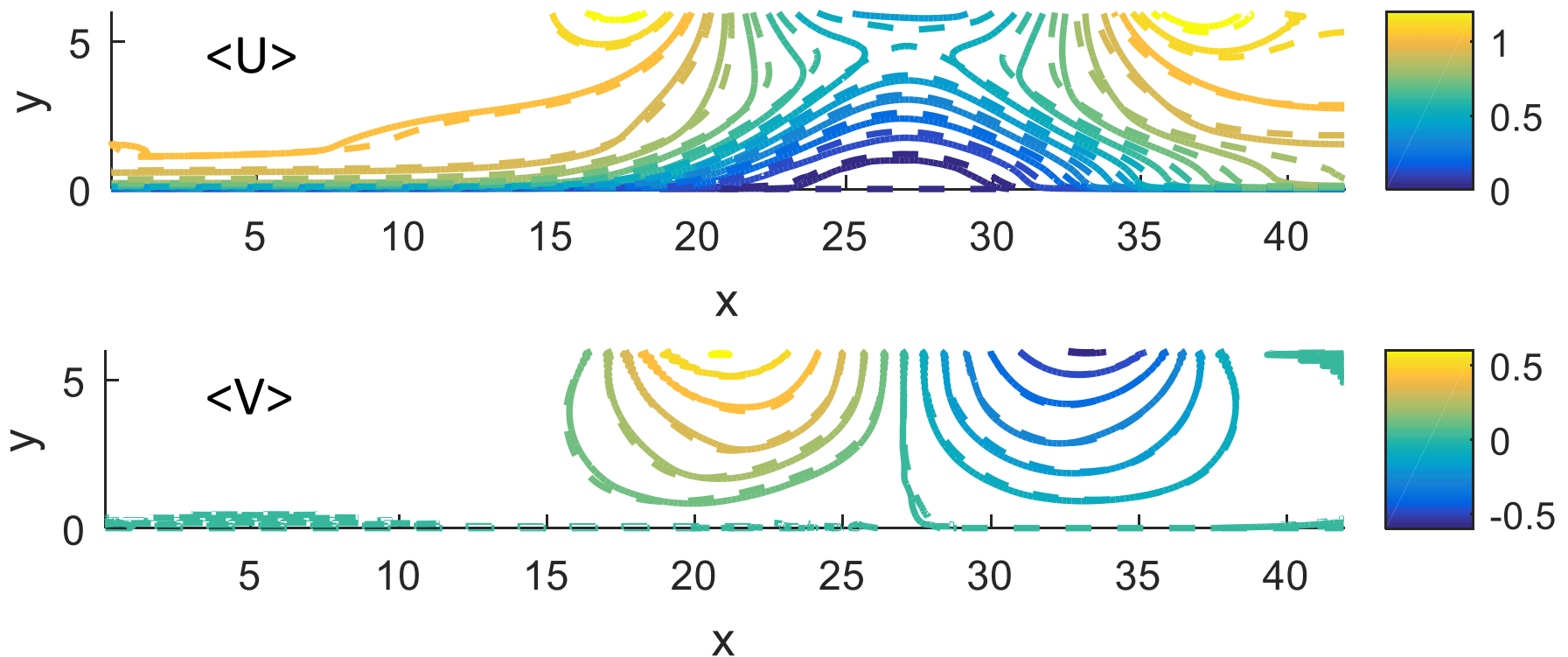
Wall-resolved LES (lines) vs iWMLES (dashes)

$$\Delta y^+ \sim 1 \qquad \Delta y^+ \sim 16, \qquad \Delta_y^+ \sim 100$$

Wall-resolved LES (lines) vs iWMLES (dashes)

$\Delta y^+ \sim 1$ $\Delta y^+ \sim 16,$ $\Delta_y^+ \sim 100$

Profiles are NOT normalized
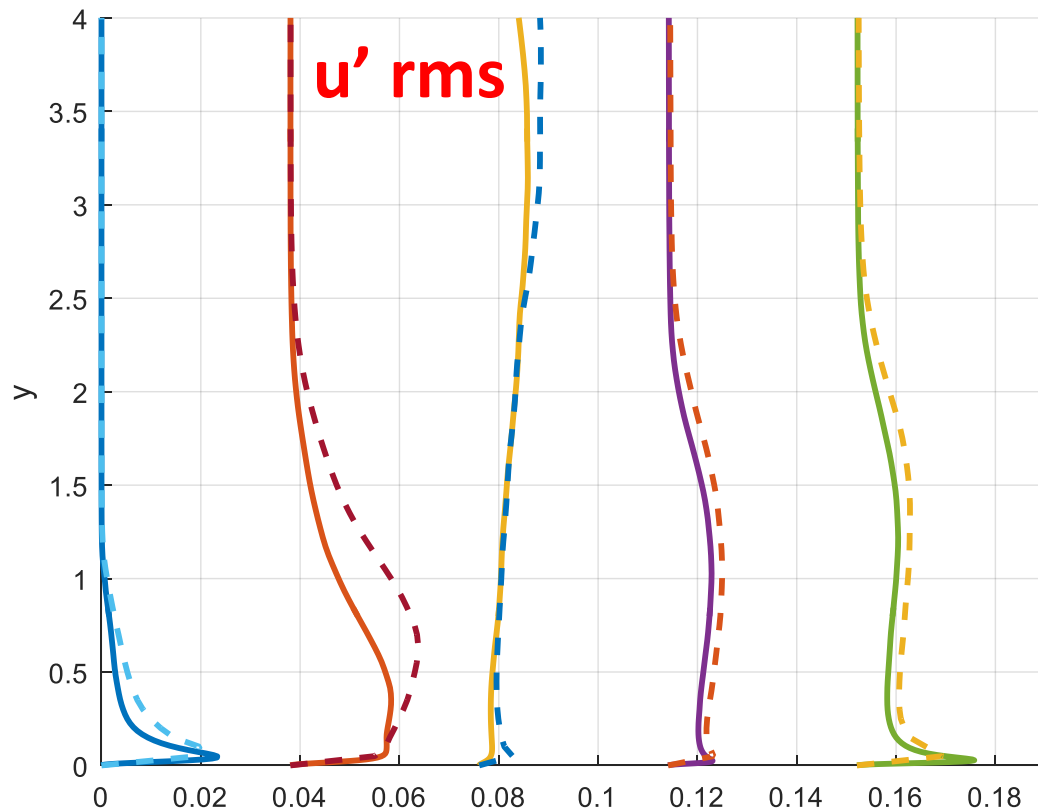
inflow   separated region   after reattachment

Wall-resolved LES (lines) vs iWMLES (dashes)

$\Delta y^+ \sim 1$ $\Delta y^+ \sim 16,$ $\Delta_y^+ \sim 100$
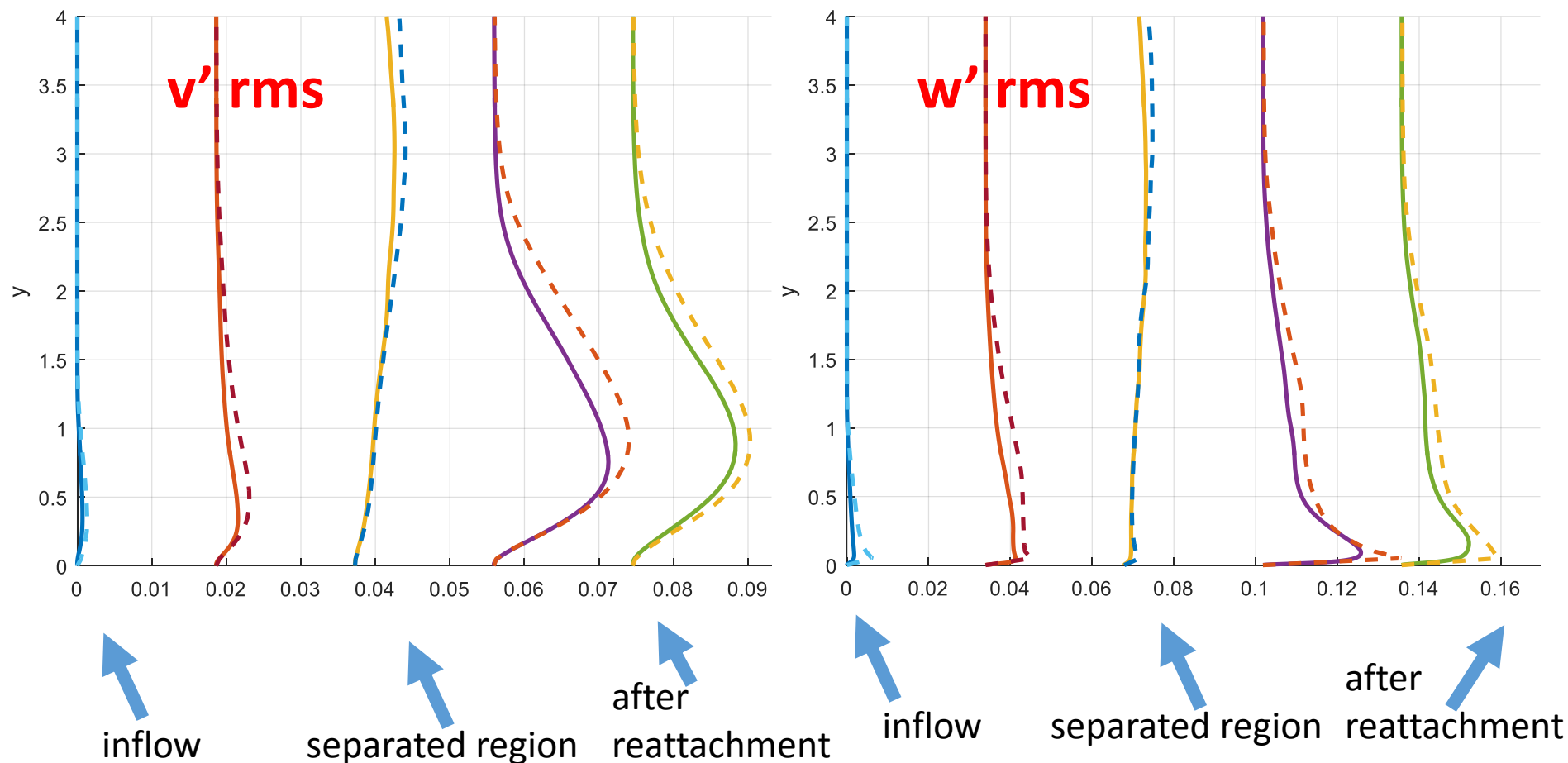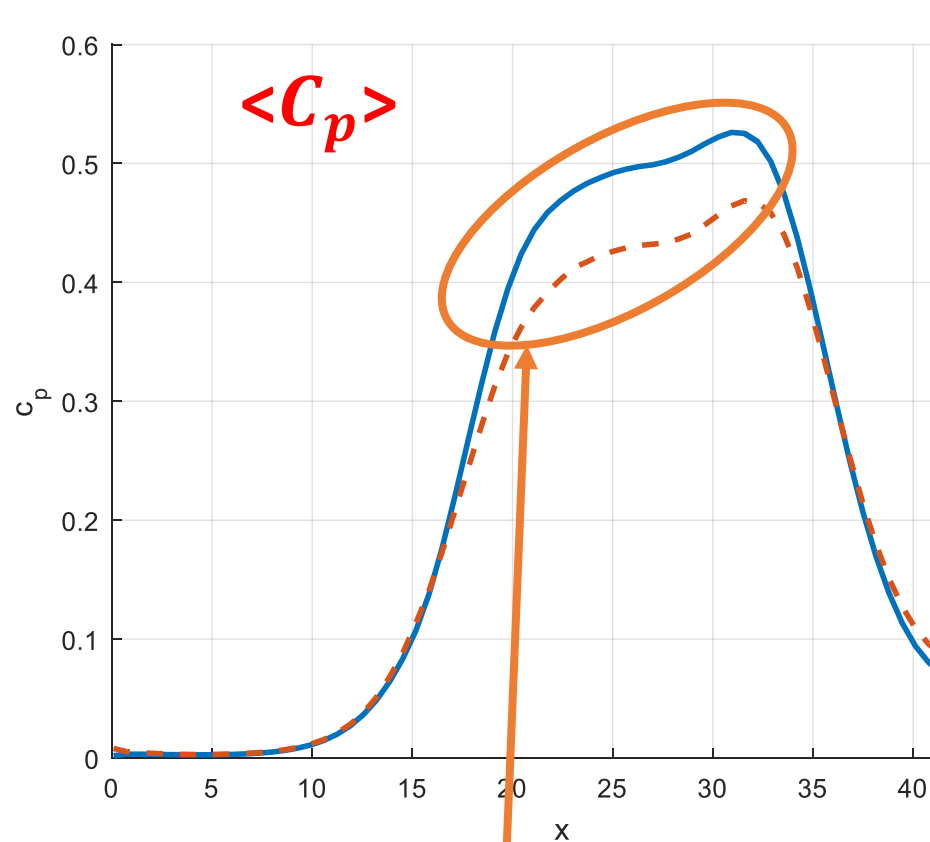

u' rms

Profiles are NOT normalized

inflow   separated region   after reattachment

27

Wall-resolved LES (lines) vs iWMLES (dashes)

$\Delta y^+ \sim 1$ $\Delta y^+ \sim 16$, $\Delta_y^+ \sim 100$

v' rms

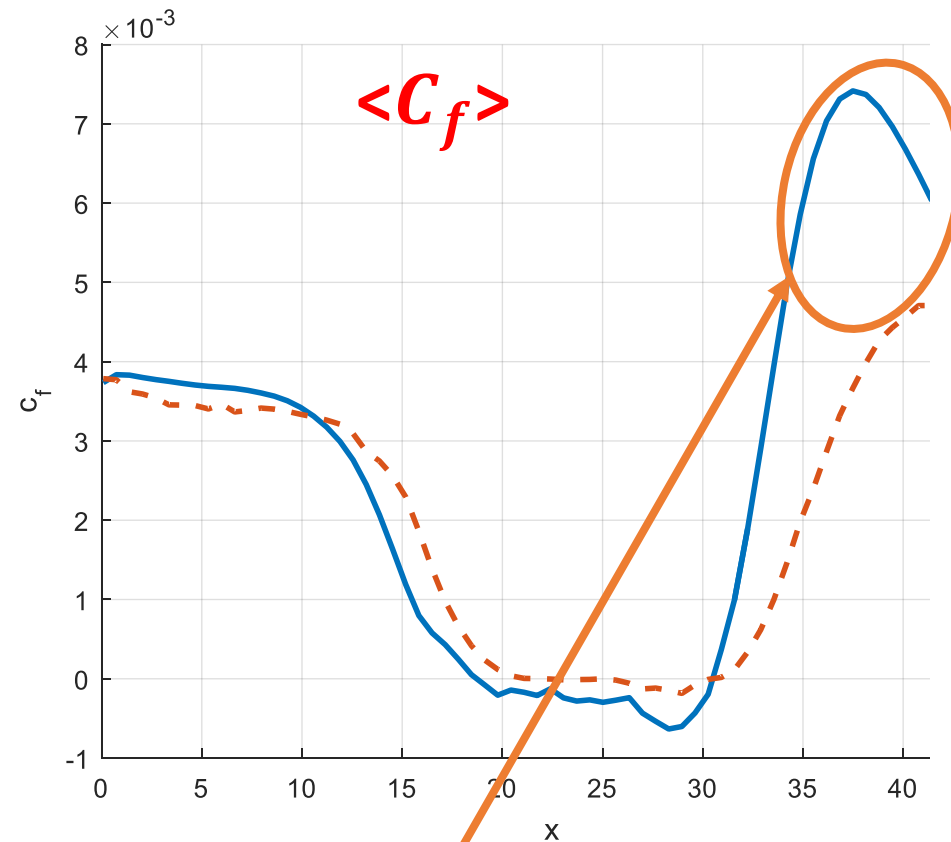w' rms

inflow   separated region   after reattachment

inflow   separated region   after reattachment

28

Wall-resolved LES (lines) vs iWMLES (dashes)

$$\Delta y^+ \sim 1 \qquad\qquad \Delta y^+ \sim 16, \qquad \Delta_y^+ \sim 100$$

Peak Cp deficit: possibly due to higher w' in iWMLES inflow, shielding near-wall

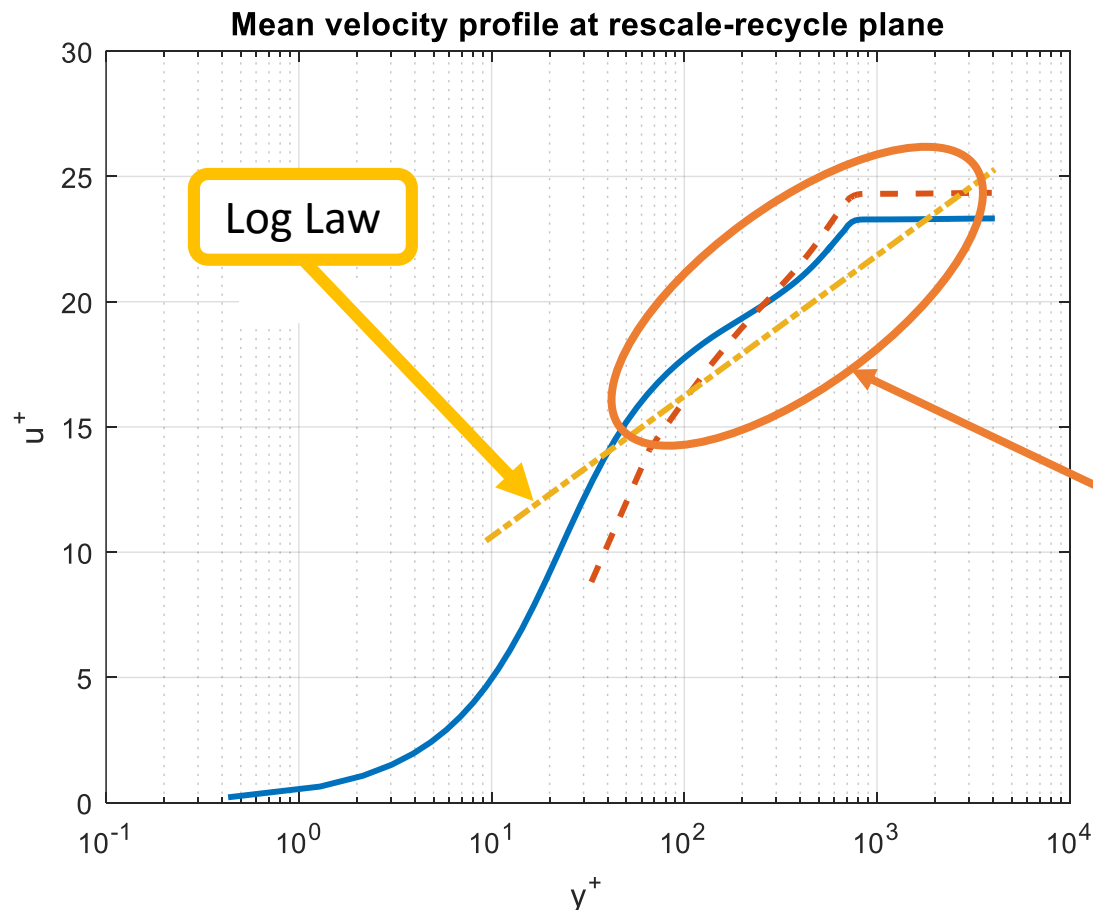Peak Cf overshoot: sign of LES under-resolution in spanwise, streamwise direction

Wall-resolved LES (lines) vs iWMLES (dashes)

$\Delta y^+ \sim 1$ $\qquad\qquad \Delta y^+ \sim 16,$ $\qquad \Delta_y^+ \sim 100$
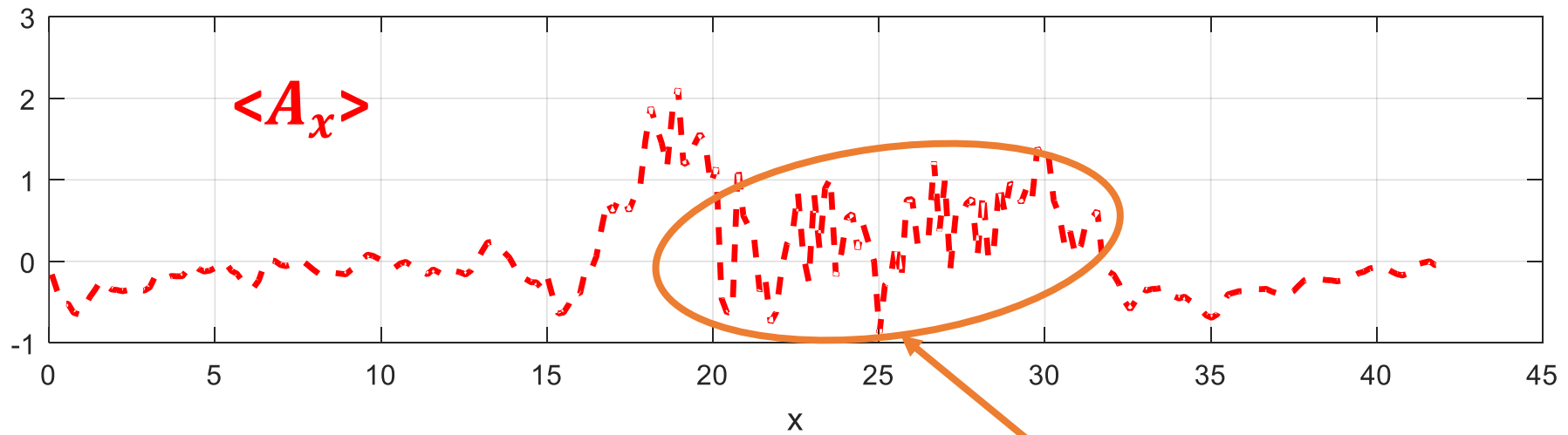
Mean velocity profile at rescale-recycle plane

Log Law

iWMLES disagreement with log-law at 'inflow' could be an indication of coupling of WM and recycle-rescale method

30

iWMLES Influence of non-equilibrium terms

$$u = u_\nu \left[\frac{y}{\delta_\nu} \right]$$

$$u = u_\tau \left[C + \frac{1}{\kappa} \log\left(\frac{y}{\Delta_y} \right) + A_x \frac{y}{\Delta_y} \right]$$

Currently analyzing strong fluctuations in A to refine numerical treatment of wall-model in ViCar3D

31

Conclusions

- Proposed a low-cost non-equilibrium integral Wall Model for LES (iWMLES)

- Validated iWMLES for canonical turbulent BL and wall-mounted cubes in turbulent channel flow

- Demonstrated iWMLES capability to predict separation, transition and reattachment for a laminar separation bubble flow

- Showed preliminary, but promising comparison of iWMLES to wall-resolved LES for a turbulent separating and reattaching boundary layer

Outlook

- Ongoing: validation of turbulent recirculation zone over flat plate
  - Increase resolution in streamwise and spanwise
  - Address inflow recycling problem

- Next: perform validation for turbulent flow over airfoil against experimental data

- Future: use iWMLES to investigate active flow control to reattach flow over wing-flap or tail-rudder at operating Reynolds number

Thank You

Questions?

Acknowledgments

iWMLES <U> + Q-criterion

$\Delta y^+ \sim 16,$ $\Delta_y^+ \sim 100$